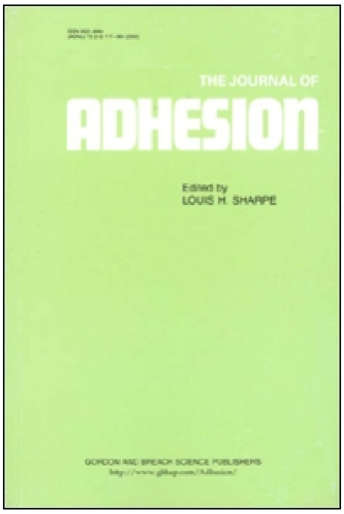

The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

Geometrically Non-Linear Analysis of Adhesively Bonded Double Containment Corner Joints

M. Kemal Apalak[a]
[a] Department of Mechanical Engineering, University of Erciyes, Kayseri, Turkey

To cite this Article Apalak, M. Kemal(1998) 'Geometrically Non-Linear Analysis of Adhesively Bonded Double Containment Corner Joints', The Journal of Adhesion, 66: 1, 117 — 133
To link to this Article: DOI: 10.1080/00218469808009962
URL: http://dx.doi.org/10.1080/00218469808009962

J. Adhesion, 1998, Vol. 66, pp. 117–133
Reprints available directly from the publisher
Photocopying permitted by license only

Geometrically Non-Linear Analysis of Adhesively Bonded Double Containment Corner Joints

M. KEMAL APALAK

Department of Mechanical Engineering, University of Erciyes, Kayseri, 38039, Turkey

(Received 17 March 1997; In final form 3 September 1997)

In cases where adhesively bonded joints may experience large displacements and rotations whilst the strains remain small, although all joint members behave elastically the small strain-small displacement (SSSD) theory cannot correctly predict the stresses and deformations in the adhesive joint members. Previous studies have shown that the small strain-large displacement theory considering the non-linear effects of the large displacements in the stresses and deformations has to be used in the analysis of adhesively bonded joints. In this study, the geometrical non-linear analysis of an adhesively bonded double containment corner joint was carried out using the incremental finite element method based on the small strain-large displacement (SSLD) theory. The objective of the study was to determine the effects of the large displacements on the adhesive and adherend stresses of the corner joint. Therefore, the corner joint was analysed for two different loading conditions; a compressive applied load, P_x, at the free end of the horizontal plate and one normal to the plane of the horizontal plate, P_y. The plates, support and adhesive layer were assumed to have elastic properties. In practice, the adhesive accumulations, called spew fillets, arising around the adhesive free ends were taken into account in the analysis since their presence results in a considerable decrease in the peak stresses around the free ends of the adhesive. The SSLD and SSSD analyses showed that the stress concentrations occurred around the free end of the adhesive, thus at the adherend (slot) corners inside the right vertical and the lower horizontal adhesive fillets, and inside the left vertical and the upper horizontal adhesive fillets for the loading conditions P_x and P_y, respectively. In addition, the plate regions around the adherend (slot) free ends along the outer fibres of the vertical and horizontal plates undergo very high stress concentrations. The SSLD analysis predicted a non-linear effect in the displacement and stress variations at the critical adhesive and plate locations, whereas the SSSD analysis showed their variations were lower and proportional to the applied incremental load. This non-linear effect became more evident for the loading condition P_x, whereas both analyses predicted very close displacement and stress variations in the adhesive fillets and in the horizontal plate for the loading condition P_y. As a result, the geometrical non-linear behaviour of the corner joint is strictly dependent on the loading condition and the large displacements affect the stress and deformation states in the joint members, and result in higher stresses than those predicted by the SSSD theory.

Keywords: Epoxy adhesives; steel adherends; containment corner joints; geometrical non-linearity; stress analysis; finite element method; adhesive fillet

1. INTRODUCTION

Because of intensive research in polymer chemistry during past decades, many new polymeric materials have been developed. Polymeric and composite mateials are used in engineering structures by means of advanced synthetic adhesives. The adhesive bonding technique is applied in the bonding of structural components with similar/dissimilar material properties or in applications in which conventional fastening methods present some disadvantages, *i.e.*, composite materials. In bonded metal assemblies, a correct design and suitable adhesive can provide great strength [1].

The analysis of adhesively bonded joints under different loading conditions has become a research area, and a large number of theoretical and experimental studies have been done [2, 3]. Generally, the stress and deformation states of the adhesive layer have been investigated. Since stress concentrations occur around the free ends of the adhesive layers, the effects of overlap length, adhesive thickness, adhesive spew fillets, geometry of the adherend edge (tapering adherend edge) in reducing these peak adhesive stresses, called the edge effect, have been analysed. In addition, considering the non-linear material properties of the adhesive and adherends, generally single-, double-lap or tubular joints have been analysed [2–4].

The finite element method, which is a very powerful numerical method described by Zienkiewicz [5], has also found extensive use in the analysis of adhesively bonded joints since the structures can be analysed regardless of their complicated geometry, non-linear material properties, loading and boundary conditions. The single-, double-lap, butt and tubular joints were analysed extensively using FEM [2, 3]. Most of these studies assumed that the adherends and adhesive had linear elastic properties, and they used the small strain-small displacement thoery.

Most engineering structures may undergo large displacements and rotations under different loading and boundary conditions. Therefore, the small strain-small displacement theory cannot predict accurately the stresses and deformations since the displacements are no longer proportional to the load. The small strain-large displacement theory can be applied to develop a solution to these types of problems [6]. The incremental finite element method based on this theory generally requires iterative solutions of a large member of non-linear equilibrium equations [5]. However, many parameters neglected in the analytical approach can be taken into account and a more realistic solution of the problem can be achieved. Wood *et al.*, Stricklin and Haisler, Carey, Bath and Cimento are among those who have contributed to the development of the incremental finite element method including geometrical non-linear effects [7–11]. More details can be found in Refs. [5, 12–13].

Sawyer and Cooper investigated the load transfer of a single lap joint by considering that the dependence of the moment on the applied load makes the problem geometrically non-linear [14]. They found that pre-forming the adherends reduced the moment resultant in the adherend at the edge of the overlap region, which causes a reduction in both the peeling and shearing stresses and gives a more uniform shear stress distribution in the adhesive layer. In order to predict the failure mode and of single- and double-lap joints having adherends and adhesives with different mechanical properties, Adams *et al.* used a non-linear finite element technique which was able to account for the large displacements and rotations and allowed for the effects of non-linear material behaviour of both adhesive and adherends [2, 15]. They found that the mechanical properties of adhesive and adherends have a considerable effect on the failure mode and loads, and that modifying the geometry of the double lap joint in the critical regions at the edge of the overlap causes significant increases in the joint strength. Reedy and Roy, Czarnocki and Pierkaski, Edlund and Klarbring have also contributed to the development of a general analysis method, allowing geometrical non-linearity, for the determination of the adhesive and adherend stresses and deformations in adhesively bonded joints [16–19].

2. JOINT CONFIGURATION

In bonding applications, a common problem occurs when joining two plates of the same or different materials at right angles each other. Due to joint geometry and loading conditions, peel and cleavage stresses generally arise. This problem can be solved by shaping one or both of the plates or providing additional support in such a manner as to minimise these stresses and ensure that most of the bonded area is under compression. Based on the last approach, Davies and Khalil presented a double containment corner (DCCR) joint whose plates are bonded into slots of a double containment corner support [20]. They carried out the stiffness analysis of this joint and determined the geometrical parameters affecting its stiffness relative to the plate thickness. Although they used a poor mesh, especially around the free ends of the adhesive and neglected the presence of the adhesive fillets, they showed that the DCCR joint is an unbalanced joint. Later, Apalak *et al.* carried out the stress and stiffness analyses of this DCCR joint and its modifications using the linear elastic finite element method based on the small strain-small displacement theory for different loading conditions [21–23]. They used an improved mesh and considered the adhesive accumulations around the free ends of the adhesive. They found that the most critical adhesive regions were the free ends of the support-adhesive-plate interfaces and the adhesively bonded corner joints exhibited a different behaviour for each loading condition. They also showed that the main parameters affecting the stress and deformation states of the corner joints were support length and slot depth and that these joints were very rigid, especially in the support region.

In this study, the geometrically non-linear analysis of an adhesively bonded double containment corner joint shown in Figure 1. was carried out using the non-linear finite element method based on the small strain-large displacement theory. The joint consists of a chamfered double containment support, a horizontal plate, a vertical plate and an adhesive layer. Since this joint has an unbalanced geometry and loading conditions, high stress concentrations are induced by the high bending moment around the free ends of the adhesive layer-adherend interface.

In the analysis, considering the joint dimensions advised by Apalak *et al.*, a bonded double containment corner joint was used having a containment support length, **a**, of 15.6 mm, slot depth, **b**, of 6 mm, adhesive thickness, δ, of 0.3 mm, horizontal and vertical plate thickness, **t**, and support thickness, **c**, of 3 mm, joint length, **L**, of 60 mm and joint width, **W**, of 500 mm [21]. These dimensions of the

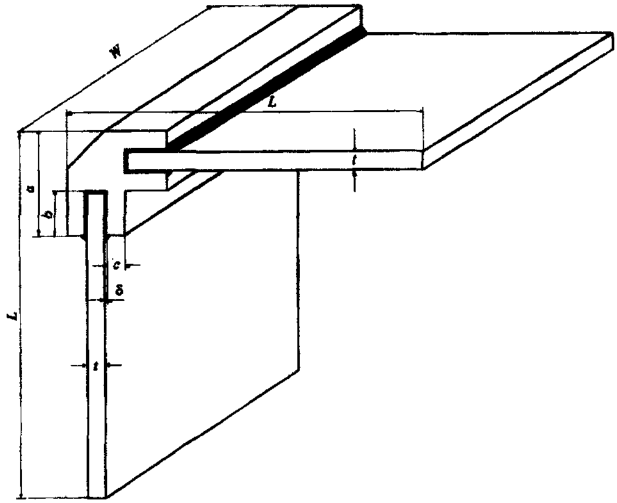

FIGURE 1 Geometry and dimensions of an adhesively bonded double containment corner joint.

joint were kept constant throughout the study. Since the geometry along the width of the double containment cantilever joint is uniform and the applied loads do not change in that direction, the problem can be reduced to one of plane strain.

The double containment joint was fixed by giving zero displacements in the x- and y-directions at the nodes along the bottom of its vertical plate as shown in Figure 2, and was analysed for two loading conditions: a load, P_y, of 20 kN applied at the end of the horizontal plate in the normal direction to the plane of the horizontal plate and a compressive load, P_x, of 20 kN applied at the uppermost node at the free edge of the horizontal plate. An epoxy-based adhesive having modulus of elasticity $E_a = 3.33$ GPa and Poisson's ratio $\nu_a = 0.34$ was used to bond the double containment support and the vertical and horizontal plates made of steel having a modulus of elasticity $E = 210$ GPa and Poisson's ratio $\nu = 0.29$. The materials of all joint members *i.e.*, plates, support and adhesive, were assumed to have linear elastic properties.

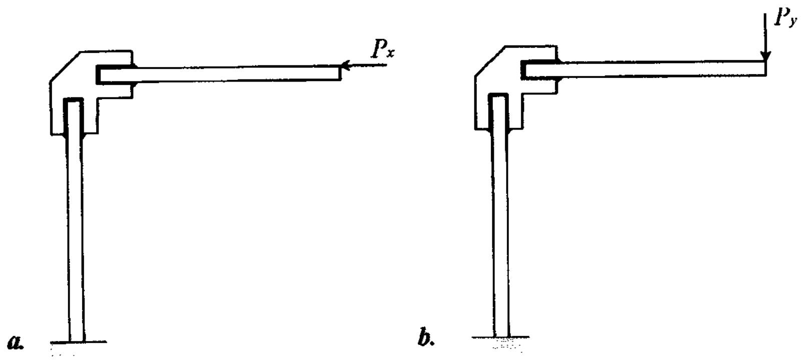

FIGURE 2 Boundary and loading conditions of the adhesively bonded double containment corner joint.

Analytical and photo-elastic studies of adhesively bonded joints have shown that the adhesive accumulation around the adhesive free ends, called spew fillet, has a considerable effect on the peak adhesive stresses and strains, and increasing the adhesive fillet size reduces the peak stresses and strains [2, 24–25]. Apalak *et al.* also have shown that these fillets have a similar effect on the stress concentrations around the free ends of the adhesive in their study in which the analysis of an adhesively bonded DCCR joint was carried out [21]. Therefore, the adhesive fillets were considered in the SSLD analysis of the DCCR joint, and their shape was idealised to a triangle of a height and width twice the adhesive thickness due to ease of meshing them.

Generally, the finite element method is applied to any continuous medium under a given boundary and loading conditions by dividing the continuum into elements with finite size including nodes at its corners and edges. Therefore, eight-noded isoparametric quadratic quadrilateral plane elements with four integration points were used to model the vertical and horizontal plates, double containment support and adhesive layer. In addition, six-noded isoparametric quadratic triangular plane elements with three integration points were used to model the joint regions in which the use of the other element type is not possible, *i.e.*, transition regions between mesh areas and in the adhesive fillets. A series of analyses have shown that mesh refinement, particularly free ends of adhesive in which high stress and strain gradients occurred, is necessary in order to achieve reasonable results. The free ends of the adhesive were divided into eight elements across the adhesive thickness while the other adhesive regions were divided into three elements as shown in Figure 3.

3. ANALYSIS AND RESULTS

The deformation and stress states of the adhesively bonded joints have been shown to be dependent of the boundary and loading conditions [1 – 3]. Therefore, the geometrical non-linear analysis of the adhesively bonded double containment corner (DCCR) joint was carried out for two loading conditions as shown in Figure 2. The main dimensions of

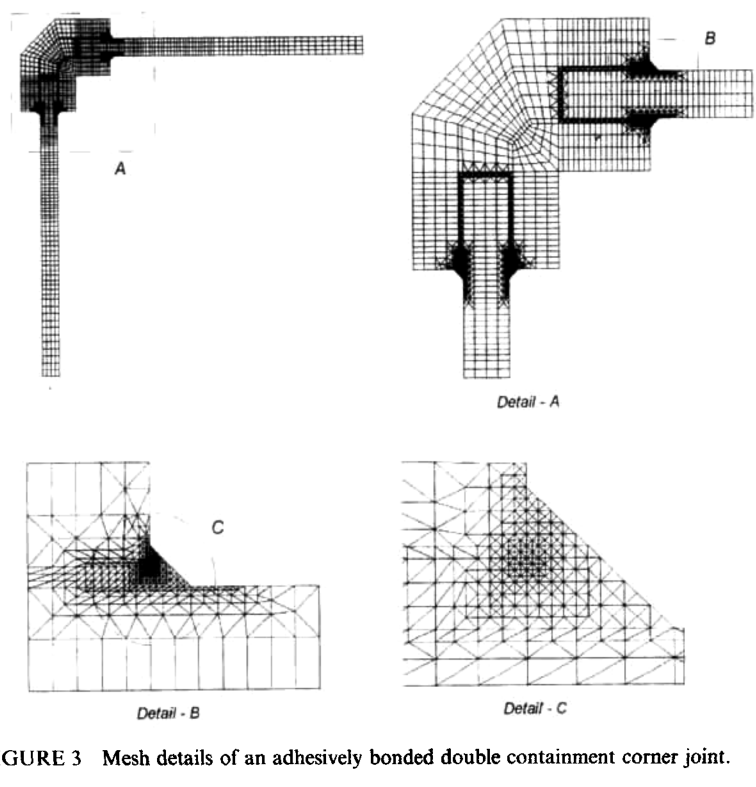

FIGURE 3 Mesh details of an adhesively bonded double containment corner joint.

the DCCR joint were determined as recommended by Apalak *et al.* [21]. The geometrically non-linear analyses of the DCCR joint considering the large displacement effects yielded a good solution with convergence values of 0.01% and 0.005% for the loading conditions P_x and P_y respectively.

The stress and strain concentrations occurred around the free ends of the adhesive layers. The detailed analyses showed that the stresses became a maximum at the adherend corners at the free ends of the adhesive-horizontal plate interfaces and of the adhesive-vertical plate interfaces. The vertical and horizontal plates were also subjected to high stresses at the plate regions corresponding to the free ends of the adhesive for both loading conditions. The detailed analysis of these critical regions was given in Ref. [21].

First, the small strain-large displacement (SSLD) analysis of the DCCR joint was carried out for the compressive horizontal loading, P_x, applied at the free end of the horizontal plate. The stress distributions in the lower and upper horizontal fillets and in the left and right vertical adhesive fillets at the slot free ends were examined. The SSLD analysis showed that the peak stresses occurred at the slot corners inside the adhesive fillets. When the stress distribution inside the left vertical adhesive fillet was compared with that inside the right vertical adhesive fillet, it was found that they were similar; however, whereas the right vertical adhesive fillet was under tension, the left vertical fillet was under compression. In addition, the lower and upper horizontal adhesive fillets presented similar stress distributions. Whereas the lower horizontal adhesive fillet was under tension the upper horizontal adhesive fillet was under compression. Since tensile stresses play an important role in the strength of the bonded joints, and because the right vertical adhesive fillet and the lower horizontal adhesive fillet were under tension, they behaved as the critical adhesive regions of the DCCR joint, in which the probable first failure could be expected for the loading condition P_x. In addition, the small strain-small displacement (SSSD) analysis of the DCCR joint was carried out in order to compare the results of both the SSSD and SSLD analyses. The compressive load, P_x, of 20 kN was applied in a total of 100 steps.

The Von Mises stresses at the integration (Gauss) points (*A* and *P*) of the adhesive elements closest to the support corners (Fig. 4) inside the right vertical and the lower horizontal adhesive fillets were

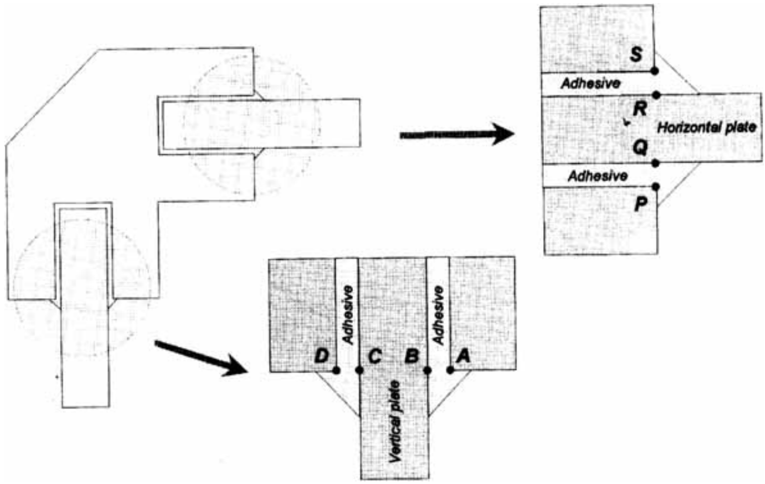

FIGURE 4 Critical adhesive and plate locations of an adhesively bonded double containment corner joint.

determined for each load increment, then their variations were plotted *versus* the increasing load in Figure 5a. The SSLD analysis predicted a non-linear effect in the variations of the Von Mises stresses in both critical points (A and P) in the right vertical and the lower horizontal adhesive fillets. The SSSD analysis showed that the Von Mises stresses were lower and proportional to the applied load. Thus, the Von Mises stresses based on the SSLD analysis are higher by 38% and 433% at the critical points A and P, respectively. It is clear that the Von Mises stresses at the critical adhesive locations are affected by the large displacements for the loading condition P_x, thus, they present a non-linear variation and reach much higher values than those predicted by the SSSD analysis. In addition, when the Von Mises stresses at the critical points (A and P) based on the SSLD analysis are compared, the stresses at the critical point A inside the right vertical adhesive fillet are higher by 246% than those at the critical point P inside the lower horizontal adhesive fillet. Therefore, the right vertical adhesive fillet is the most critical adhesive region for the loading condition P_x.

Based on the small strain-large displacement analysis, the Von Mises stresses in all critical adhesive regions of the adhesively bonded

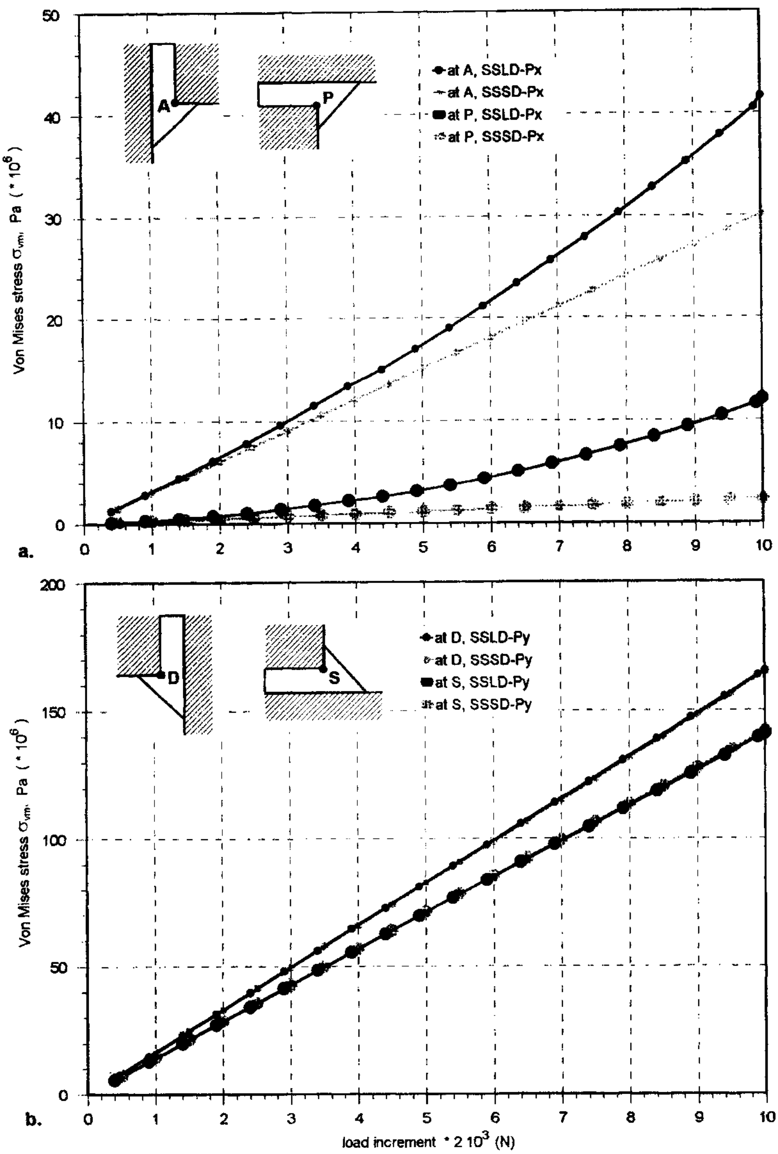

FIGURE 5 Von Mises stresses at the critical points a) A and P (Fig. 4) inside the right vertical and lower horizontal adhesive fillets, and b) D and S (Fig. 4) inside the left vertical and upper horizontal adhesive fillets of a double containment corner joint for the loadings P_x and P_y respectively, based on both the small strain-small displacement (SSSD) and the small strain-large displacement (SSLD) analyses.

double containment corner joint presented high non-linearity. It is evident that consideration of large displacements and rotations has an important effect in predicting correctly the stress and deformation states in the members of this adhesively bonded joint. However, these results were obtained for only the loading condition P_x, and whether another loading condition results in a similar behaviour in the members of the DCCR joint is an unanswered question.

For this purpose, the DCCR joint was analysed under the second loading condition P_y applied in the normal direction to the plane of the plate at the free end of the horizontal plate as shown in Figure 2b. The loading, P_y, of 20 kN was applied in a total of 100 steps. The SSLD and SSSD analyses showed that the stress and strain concentrations occurred around the free ends of the adhesive corresponding to the horizontal and vertical free ends of the slot and became a maximum at the slot corners inside the adhesive fillets. Since the stress components are tensile at the slot corners inside the left vertical and the upper horizontal adhesive fillets, these adhesive fillets were considered.

The Von Mises stresses at the integration (Gauss) points (D and S) of the adhesive elements corresponding to the support corners (Fig. 4) inside the left vertical and the upper horizontal adhesive fillets were determined for each load increment, then their variations were plotted *versus* the increasing load in Figure 5b. Both SSLD and SSSD analyses showed that the Von Mises stresses were very close and proportional to the applied load. Contrary to the loading condition P_x, the Von Mises stresses at the critical adhesive locations do not present any non-linear variation for this loading condition. Both analyses also showed that the Von Mises stresses at the critical point D inside the left vertical adhesive fillet were higher by 17% than those at the critical point S inside the upper horizontal adhesive fillet. Therefore, the left vertical adhesive fillet is the most critical adhesive region of the DCCR joint for the loading condition P_y. As seen, large displacements do not occur in the critical adhesive regions of the DCCR joint for any loading condition; consequently, their non-linear effect on the stress variations is not observed. Finally, the Von Mises stresses occurring at the critical point D inside the left vertical adhesive fillet, being the most critical adhesive region for the loading condition P_y (Fig. 5b), are higher by 297% than those at the critical point A inside the right vertical adhesive fillet, being the most critical adhesive region for the loading condition P_x (Fig. 5a).

Generally, the adhesive layer in the bonded joints behaved as a weak link. However, in case adhesives with high strength are used to bond the adherends, metal yielding can also be expected in the adherends. Therefore, the stress and deformation states of the adherends should be analysed carefully. The detailed analysis of the stresses and strains in the vertical and horizontal plates showed that both plates were subjected to high stress distributions along their outer fibres for the loading conditon P_x, and that the stresses concentrated on the critical plate regions corresponding to the vertical and horizontal free ends of the slot (C and Q in Fig. 4).

Since the Von Mises stresses at the critical location (C in Fig. 4) on the left outer fibre of the vertical plate corresponding to the free end of the vertical slot exhibit higher values than those at the critical point (B in Fig. 4) on the right outer fibre of the vertical plate, the variations of the Von Mises stresses at the critical point C on the vertical plate were plotted *versus* the incremental load in Figure 6a based on the SSLD and SSSD analyses. Whereas the SSLD analysis predicts a non-linear variation for the Von Mises stresses at the critical point C, the SSSD analysis predicts the Von Mises stress variation being lower and proportional to the applied load. Thus, the Von Mises stresses at the critical point C based on the SSLD analysis are higher by 14% than those predicted by the SSSD analysis as shown in Figure 6a.

In addition, the stresses reached a maximum at the second critical location (Q in Fig. 4) along the lower outer fibre of the horizontal plate corresponding to the free end of the horizontal slot. In order to compare the results of the SSSD and SSLD analyses, the Von Mises stress variations at this critical location were plotted *versus* the incremental load as shown in Figure 6a. Based on the SSLD analysis, the Von Mises stresses at the critical point Q varied non-linearly and were higher by 300% than those predicted by the SSSD analysis. In addition, the stresses based on the SSSD analysis were proportional to the applied load.

When the Von Mises stresses at the critical locations (C and Q) in the vertical and horizontal plates are compared, the Von Mises stresses at the critical point C on the vertical plate were higher by 647% than those at the critical location Q on the horizontal plate for the loading

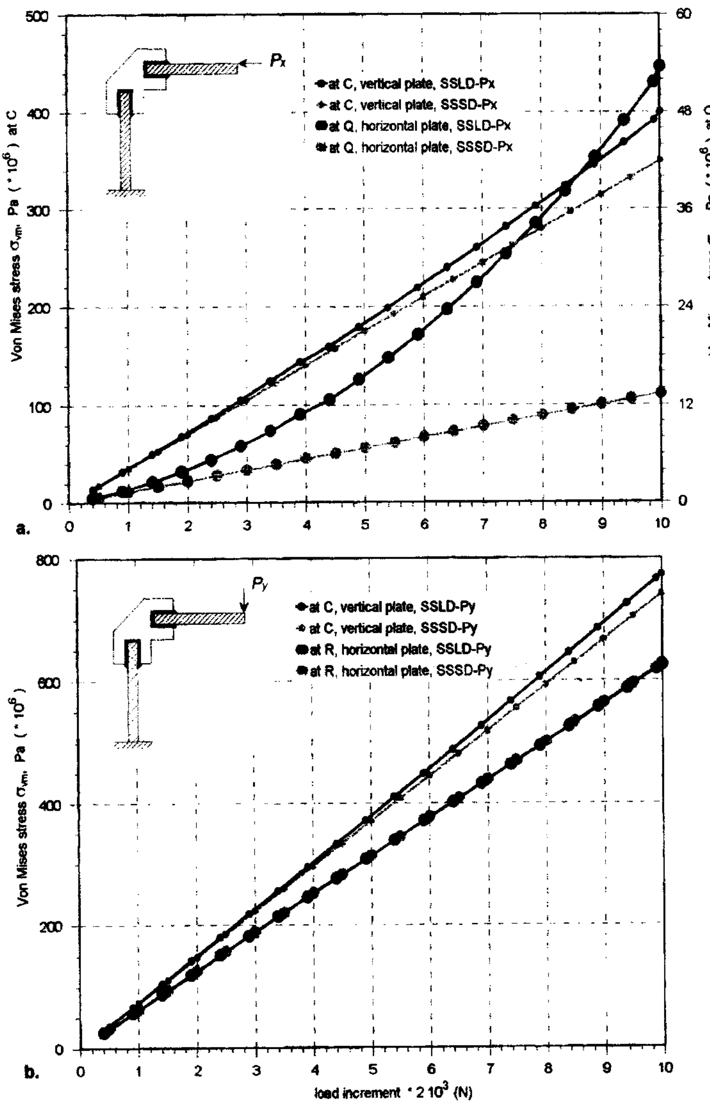

FIGURE 6 Von Mises stresses at the critical points a) C and Q (Fig. 4) at the vertical and horizontal plates, and b) C and R (Fig. 4) at the vertical and horizontal plates of a double containment corner joint for the loading conditions P_x and P_y, respectively, based on both the small strain-small displacement (SSSD) and the small strain-large displacement (SSLD) analyses.

condition P_x. Therefore, the vertical plate, especially its critical location Q, is the most critical adherend location of the DCCR joint for the loading condition P_x. In addition, it is evident that large displacements result in the Von Mises stresses having higher non-linear variation.

In case of the loading condition P_y high stress distributions occurred along the outer fibres of the vertical and horizontal plates, and the stresses concentrated on the regions of the vertical and horizontal plates corresponding to the free ends of the vertical and horizontal slots. However, the Von Mises stresses reached peak values at the critical locations C and R (Fig. 4) of the vertical and horizontal plates, respectively. The variations of the Von Mises stresses at these critical points (C and R) were determined based on the SSLD and SSSD analyses, and plotted *versus* the applied load in Figure 6b. The SSLD and SSSD analyses showed that the Von Mises stresses at the critical points C and R on the vertical and horizontal plates had very similar variations and were proportional to the applied load. The Von Mises stresses predicted by the SSLD analysis were higher by 4.3% and 2.5% at the critical points C and R, respectively, than those predicted by the SSSD analysis. In addition, when the Von Mises stresses at the critical points C and R on the vertical and horizontal plates are compared, it is evident that the stresses at the critical location C on the vertical plate are higher for the loading condition P_y. However, the Von Mises stresses at the critical point R are still high; therefore, both vertical and horizontal plates can be assumed to be the most critical members of the DCCR joint for this loading condition.

Finally, the loading condition plays a very important role in the stress and deformation states of an adhesively bonded double containment corner joint. Thus, the geometrical non-linearity occurs as a result of the large displacements and rotations; consequently, the stress and deformation states of the DCCR joint are considerably affected by this non-linearity. When the plate thickness is sufficiently small so that the large displacements and rotations occur without causing any plastic deformation, the small strain-small displacement theory cannot predict accurately the stress and deformation states of the adhesively bonded joint. Thus, the SSSD theory predicts much lower stress and deformation values than those in fact. The SSLD analysis of the DCCR joint showed that the stress values in the critical regions of the plates and the adhesive layer were higher for the loading conditions P_y.

4. CONCLUSIONS

In this study, the adhesively bonded double containment corner joint was analysed using the finite element method based on the small strain-small displacement, and the small strain-large displacement theories for two different loads. Both analyses showed that the adhesive stresses concentrated around the vertical and horizontal free ends of the slot and that they became a maximum at the slot corners inside the adhesive fillets for all loading conditions. In addition, the vertical and horizontal plates had critical regions subjected to high stresses corresponding to the free ends of the slot.

In case of the loading condition P_x, the SSLD theory showed that the Von Mises stresses in the vertical and horizontal adhesive fillets had non-linear variations and that they were higher than those predicted by the SSSD theory, whereas the SSSD theory predicted that they became proportional to the increasing load. The right vertical adhesive fillet appeared as the most critical adhesive region for the loading condition P_x. In the vertical and horizontal plates, the peak stresses occurred at the outer fibres of both plates corresponding to the free ends of the slot. The Von Mises stresses based on the SSLD theory in the vertical plate presented a non-linear variation, and were considerably higher than those predicted by the SSSD analysis. The non-linear effect in the Von Mises stress variations became more evident in the horizontal plate, and both theories predicted that the right vertical adhesive fillet and the vertical plate are the most critical regions of the DCCR joint for the loading conditions P_x.

In case of the loading condition P_y, the SSSD and SSLD analyses showed the Von Mises stresses at the slot corners inside the vertical and horizontal adhesive fillets had very similar variations and that the left vertical adhesive fillet was subjected to much higher stresses. In addition the horizontal and vertical plates had critical regions subjected to high stresses corresponding to the free ends of the slot. Whereas the SSLD theory predicted a small non-linear effect in the

Von Mises stress variations in the vertical plate, a non-linear effect was not observed in those in the horizontal plate. The left vertical adhesive fillet and the vertical plate were subjected to higher stresses for the loading condition P_y.

Finally, the adhesively bonded double containment corner joint exhibits different stress and deformation states depending on the loading condition, and large displacements play an important role in the prediction of the stress and deformation states of the adhesive and adherends. The small strain-large displacement theory can consider the non-linear effect of the large displacements. The study shows that the adhesively bonded DCCR joint may have much higher stresses and deformations than those predicted by the small strain-small displacement analysis in cases where the loading conditions result in large displacements; therefore, the SSSD approach may be completely misleading.

References

[1] Semerdjiev, S., *Metal to Metal Adhesive Bonding* (Bussiness Books Ltd., London, 1970).
[2] Adams, R. D. and Wake, W. C., *Structural Adhesive Joints in Engineering* (Elsevier Applied Science, London, 1984).
[3] Kinloch, A. J., *Adhesion and Adhesives* (Chapman and Hall, London, 1987).
[4] Hart-Smith, L. J., in *Developments in Adhesives – 2*, A. J. Kinloch, Ed. (Elsevier Applied Science Publishers, London, 1981), pp. 1–44.
[5] Zienkiewicz, O. C. and Taylor, R. C., *The Finite Element Method-Solid and Fluid Mechanics, Dynamics and Non-linearity* Vol. **2**, (McGraw-Hill Company, UK, 1991).
[6] Malvern, L. E., *Introduction to the Mechanics of a Continuous Medium*, Prentice-Hall, Inc., New York, 1969).
[7] Wood, R. D. and Zienkiewicz, O. C., *Computers & Structures* **7**, 725–735 (1977).
[8] Wood, R. D. and Schrefler, B., *Int. J. Numerical Methods in Engineering* **12**, 635–642 (1978).
[9] Stricklin, J. A. and Haisler, W. E., *Computers and Structures* **7**, 125–136 (1977).
[10] Carey, G. F., *Computer Methods in Applied Mechanics and Engineering* **4**, 67–79 (1974).
[11] Bathe, K. J. and Cimentò, A. P., *Computer Methods in Applied Mechanics and Engineering* **22**, 59–85 (1980).
[12] Crisfield, M. A., *Non-linear Finite Element Analysis of Solids and Structures* **1**, (John Wiley and Sons, 1991).
[13] Kleiber, M., *Incremental Finite Element Modelling in Non-linear Solid Mechanics* (Ellis Horwood Series in Mechanical Engineering, 1989).
[14] Sawyer, J. W. and Cooper, P. A., *AIAA Journal* **19**(11), 1443–1451 (1981).
[15] Adams, R. D., *Developments in Adhesives –2*, A. J. Kinloch Ed. (Elsevier Applied Science, London, 1981) pp. 45–81.

[16] Reddy, J. N. and Roy, S., *Int. J. Non-linear Mechanics* **23**(2), 97–112 (1988).
[17] Czarnocki, P. and Pierkaski, K., *Int. J. Adhesion and Adhesives* **6**(3), 157–160 (1986).
[18] Edlund, U. and Klarbring, A., *Computer Methods in Applied Mechanics and Engineering* **78**, 19–47 (1990).
[19] Edlund, U. and Klarbring, A., *Computer Methods in Applied Mechanics and Engineering* **96**, 329–350 (1992).
[20] Davies, R. and Khalil, A. A., *Int. J. Adhesion and Adhesives* **10**(1), 25–30 (1990).
[21] Apalak, M. K., Davies, R. and Apalak, Z. G., *J. Adhesion Sci. Technol.* **9**(2), 267–293 (1995).
[22] Davies, R., Apalak, M. K. and Apalak, Z. G., *J. Strain Anal. Eng.* **30**(2), 91–115 (1995).
[23] Apalak, M. K., Davies, R. and Apalak, Z. G., *J. Adhesion Sci. Technol.* **10**(10) 907–937 (1996).
[24] Tuzi, I. and Shimada, H., *Bull. JSME* **7**, 263–267 (1964).
[25] Tuzi, I. and Shimada, H., *Bull JSME* **8**, 330–336 (1965).